2.9 Birth control

A simple simulation might model the problem using a random number generator which produces a number x with equal probabilities of being 0 or 1. Let the outcome 1 represent the event that a boy is born. The necessary pseudocode is

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Make a family:
pick a random number x
if x = 0 add the score x to the family and continue making a family
if x = 1 add the score x to the family and stop.
Repeat for a large number of families to make a population.
Find the average score in the population.
```

This piece of code will show that the average score is 0.5, so the birth control plan does not seem to achieve its objective. However, there is no limit (in this model) on the family size. If we limit the family size, the simulation still gives an average of 0.5. Careful how you limit the size; limiting the size is not compatible with the original plan of stopping on boy births, and so a limit means there are large families of just girls. Limiting the family size is incompatible with the original strategy! These rare, large families are essential to redress balance back in favour of girls.

For an exact solution we focus on the family size n. The probability that a family will have $n-1$ girls and then a boy (in that order) is

$$\text{prob}(n) = \frac{1}{2^n}$$

and so the expected number of boys is

$$b = \sum_{n=1}^{N} \frac{1}{2^n}$$

where N is the maximum family size. For $N \to \infty$ this geometric series sums to 1, as expected.

The expected number of children is

$$c = \sum_{n=1}^{N} n \,\text{prob}(n).$$

This is not a geometric series, but it is the derivative of one, and so can be summed; we get

$$c = \frac{1}{2^N}\left(2^{N+1} - 2 - N\right).$$

From this we find the boy fraction

$$\frac{b}{c} = \frac{2^N - 1}{2^{N-1} - N - 2}$$

which tends to 0.5 for large N. For finite N this equation gives the incorrect boy fraction, because we cannot always end a family on a boy and also limit the family size a priori.